Anyon related correlations in two-dimensional Coulomb gases

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In our recent paper (Phys. Rev. B **76**, 075403 (2007)), we have applied the anyon concept to derive an approximate analytic formula for the ground state energy, which applies to two-dimensional ($2D$) Coulomb systems from the bosonic to the fermionic limit. We make use of these results here to draw attention to correlation effects for two special cases: the spin-polarized 2D fermion system and the charged anyon system close to the bosonic limit. By comparison with quantum Monte-Carlo data (for the former) and exact results obtained in the hypernetted-chain and Bogolyubov approximations (for the latter) we can conclude on correlation effects, which have their origin in the bosonic systems and come into play by using the anyon concept. To our knowledge, these correlations are not yet considered in the literature.

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The homogeneous electron gas, although it has become the testing ground of quantum mechanical many-body techniques since long time and results of such calculations can be found in textbooks (see for example [1] and [2]), does still contain yet unexpected puzzles. They are related to the correlation effects caused by the particle-particle interaction, which can be treated so far (or principally) only in approximate ways. Here we would like to draw attention to the two-dimensional ($2D$) systems, which have their prominent realizations in semiconductor heterostructures and layered metallic systems such as the cuprate superconductors. These systems attract much interest from experiment and theory due to the physics related with correlation effects [3, 4, 5].

With respect to the ground state energy in fermion systems, the Hartree-Fock (HF) approximation - originally developed for high particle densities - provides the dominating contributions even for intermediate values of the density parameter r_s. The difference between the exact ground state energy and its HF value is known as the correlation energy. For $2D$ spin-polarized fermions, it has been demonstrated by quantum Monte-Carlo (MC) calculations [6, 7], that the correlation energy may be considered essentially as a small correction to the HF energy for all values of r_s.

Calculations of the ground state energy of $2D$ Coulomb boson systems [8] have also been performed by employing MC methods. As in [6, 7] these calculations were performed only for discrete values of r_s and an interpolation was made to yield an expansion in terms of powers of r_s. At small r_s (the high density limit), this interpolation formula reproduces the ground state energy of the $2D$ Coulomb boson gas (2DCBG) obtained with the hypernetted-chain approximation [9]. The latter result follows also by using the Bogolyubov approximation [10].

In Ref. [10] we have employed the anyon concept in order to find an approximate analytic formula for the ground state energy of $2D$ Coulomb gases, which is capable of accounting for the aspects of fractional statistics and brings together bosonic and fermionic aspects of the system. This formula was derived by using the corresponding result for the harmonically confined $2D$ Coulomb anyon gas [11] and applying a regularization procedure for vanishing confinement. Fractional statistics and Coulomb interaction have been taken into account by introducing a function $f(\nu, r_s)$, which depends on both the statistics and density parameters (ν and r_s, respectively), and was determined by fitting to the ground state energy of the classical $2D$ electron crystal at very large r_s (the $2D$ Wigner crystal) and for very small r_s (the high density limit) to that of the 2DCBG and to the HF energy of spin-polarized $2D$ electrons. For the latter and at intermediate values of r_s a comparison with HF (Ref. [12]) and MC (Refs. [6, 7]) results has revealed significant deviations, which are shown here in the enlarged scale of Fig. 1. It shows the correlation energy as a function of r_s and exhibits a pronounced minimum with $\mathcal{E}_{c,min}(r_s \simeq 0.2) \sim -8\ Ry$, where Ry is Rydberg energy unit. It appears essentially due to the minimum $\mathcal{E}_{0,min}$ of the ground state energy not only for fermions ($\nu = 1$) but for all $\nu \neq 0$, which is independent of the choice of $f(\nu, r_s)$ [10]. The fitting of $f(\nu, r_s)$ to known ground state energies for high and low density limits merely determines the r_s value, at which the minimum $\mathcal{E}_{0,min}$ appears. We found also that as a function of the statistics parameter ν and close to the bosonic limit ($\nu \to 0$) the ground state energy exhibits for high densities ($r_s \to 0$) a minimum at finite ν, at which the energy diverges with

$\sim -1/r_s$ faster than the energy of the 2DCBG for $\nu = 0$ ($\sim -1/r_s^{2/3}$, Refs. [9] and [10]).

Our expression for the ground state energy per particle derived in [10] (see Eq. (33) of [10]) is entirely determined by the function (see Eq. (40) of [10])

$$f(\nu, r_s) \approx \nu^{1/2} c_0(r_s) e^{-5r_s} + \frac{c_{BG}^{3/2} r_s / c_{WC}}{1 + c_1(r_s) c_{BG}^{3/2} r_s^{1/2} / c_{WC}} + \frac{0.2 c_1(r_s) r_s^2 ln(r_s)}{1 + r_s^2} \quad (1)$$

with $c_0(r_s) = 1 + 6.9943 r_s + 22.4717 r_s^2$ and $c_1(r_s) = 1 - e^{-r_s}$. It fits to the expression of the ground state energy of the classical $2D$ electron crystal [13], $E_{WC} = -2.2122/r_s$ (for $r_s \to \infty$). For $r_s \to 0$ it reproduces the ground state energy (expressed in Ry energy units) of the 2DCBG [9, 10]

$$\mathcal{E}_0(\nu = 0, r_s \to 0) = -c_{BG} r_s^{-2/3}\,, \quad (2)$$

where $c_{BG} = \frac{2\Gamma(-\frac{4}{3})\Gamma(\frac{5}{6})}{3\sqrt{\pi}} = 1.29355$ and the HF energy [12] $E_{HF} = 2/r_s^2 - 16/3(\pi r_s)$ for spin-polarized $2D$ electrons. In Eq. (1) we used a constant $c_{WC}^{2/3} = 2.2122$.

As seen from Fig. 1 and Fig. 2 of Ref. [10] for the interval $0.7 \leq r_s < \infty$, the ground state energy per particle (in Ry) can be well described by the formula

$$\mathcal{E}_0(\nu, r_s \to \infty) = \frac{c_{WC}^{2/3} f^{2/3}(\nu, r_s)}{r_s^{4/3}} \left(-1 + \frac{7\nu f^{2/3}(\nu, r_s)}{3 c_{WC}^{4/3} r_s^{4/3}} \right), \quad (3)$$

which for low densities ($r_s \to \infty$) or for $\nu < r_s$ is an approximate expression of the exact formula Eq. (33) of [10]. In either case, using Eq. (33) of [10] or Eq. (3) (which is Eq. (39) of [10]), the results fall below the MC data and the corresponding interpolation formula by [6]. We allocate this deviation by observing that for small r_s (but $r_s \geq 0.7$) the second term of $f(\nu, r_s)$ (Eq. (1)), which is independent of ν, becomes dominant and we may replace $f(\nu, r_s)$ by its bosonic limit $f(\nu = 0, r_s)$. Thus the dependence of the ground state energy $\mathcal{E}_0(\nu, r_s)$ on the anyon parameter ν is provided only by the factor ν in the second term of Eq. (3). Therefore, the first term of Eq. (3) with the common factor expressed by $f(0, r_s)$, which is the expression for the boson ground state energy (Eq. (36) of Ref. [10]), dominates in $\mathcal{E}_0(\nu, r_s)$, thus providing the main bosonic contribution to the fermion ground state energy for the interval $0.7 \leq r_s \leq 6$. For the interval $0 \leq r_s \leq 0.7$ the energy is expressed by exact formula Eq. (33) of [10].

In Fig. 1 we show the correlation energy for spin-polarized $2D$ fermions ($\nu = 1$) obtained from the exact formula Eq. (33) of [10] for the ground state energy by subtracting the HF energy E_{HF}. The origin of the deep minimum is connected with the second term of Eq. (1), discussed before, and can thus be ascribed to (the) bosonic correlations, which become effective in our

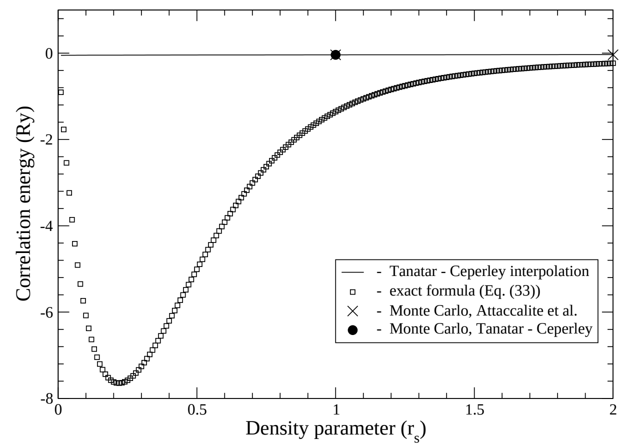

FIG. 1: Correlation energy as difference of the ground state energy (calculated from exact formula Eq. (33) of Ref. [10]) and the HF energy (open squares). MC data from Refs. [7] and [6] (cross and solid dot, respectively) and interpolation between these MC data from Ref. [6] (solid line) are given for comparison.

anyon approach and are absent in the fermionic descriptions.

The minimum of the ground state energy as function of ν found near the boson end ($\nu = 0$) of the Coulomb anyon gas occurs at $\nu_0 = b_2^{3/4} c_{WC} r_s / b_1^{1/2}$ and takes the value $\mathcal{E}_{0,min} = -(4/5) b_2^{1/4} c_{WC} b_1^{1/2} / r_s$, where $b_1 = 1 + 2.441472 r_s$ and $b_2 = 3/35$. It is derived by using Eq. (3) (because $\nu_0 < r_s$ for $r_s \to 0$) with the approximation $f(\nu, r_s \to 0) \approx \nu^{1/2} b_1$. Using the more accurate function $f(\nu, r_s \to 0) \approx \nu^{1/2}(1 + 1.99432 r_s) + 0.44713 r_s$, where $c_{BG}^{3/2}/c_{WC} = 0.44713$, does not change the expressions for $\mathcal{E}_{0,min}$ and ν_0, but one needs to replace b_1 by $b_1 = 1 + 1.99432 r_s$. In the last expression for $f(\nu, r_s \to 0)$ the first term depends on ν and describes the effect of statistics in $\mathcal{E}_{0,min}$. Without this term we had obtained (from the second term) the energy of the 2DCBG. However, if we substitute the expression for ν_0 with the new b_1 in $f(\nu, r_s \to 0)$ and take the high density limit ($r_s \to 0$) then the first term of $f(\nu, r_s \to 0)$ is always larger than the second one. This is the reason why we have a deviation from the energy of the 2DCBG. In Ref. [10] we have motivated the inclusion of the $\nu^{1/2}$ dependence in $f(\nu, r_s)$ by the linear ν dependence of the ground state energy for the anyon gas without Coulomb interaction close to bosonic limit found in [14],[15] and [16]. This linear dependence of the energy on ν is also obtained from Eq. (38) of [10] if we take the limit $r_s \to 0$ under the constraint $r_s < \nu$.

In conclusion, in the frame of our phenomenological approach based on the anyon concept, we have identified correlation effects in the high density limit of the 2D Coulomb anyon gas. In contrast with results from MC calculations for fermions, our data show a pronounced minimum in the correlation energy, which can be ascribed to bosonic correlations. Close to the bosonic limit we find at finite ν a minimum of the ground state energy, which is lower than the known value for the 2DCBG at $\nu = 0$ and is ascribed to statistical correlations. Neither of these effects is reported so far in the literature.

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